ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

IMPLEMENTING GMM-BASED HIDDEN MARKOV RANDOM FIELD FOR COLOUR IMAGE SEGMENTATION

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ABSTRACT

As it is well known to all that the terms image segmentation means dividing a picture into different types of regions and classes of particular geometric shape. So we can say that each class has normal distribution with specific mean and variance and hence the picture can be a Gaussian Mixture Model. In this paper, we first study the Gaussian-based hidden Markov random field (HMRF) model and its expectation maximization (EM) algorithm. Then we generalize it to Gaussian mixture model-based hidden Markov random field. The algorithm is implemented in MATLAB R20013a. We also apply this algorithm to color image segmentation problems.

Keywords: Image segmentation, EM algorithm, MAP Estimation, GMM-HMRF, color image segmentation

INTRODUCTION

Image segmentation is an important technology for image processing. There are many applications whether on synthesis of the objects or computer graphic images require precise segmentation. Segmentation partitions an image into distinct regions containing each pixel with similar attributes. To be meaningful and useful for image analysis and interpretation, the regions should strongly relate to depicted objects or features of interest. Meaningful segmentation is the first step from low-level image processing transforming a grey scale or color image into one or more other images to high-level image description in terms of features, objects, and scenes. Thus in general a picture can be Gaussian mixture model. In this paper, we have learned **Gaussian mixture model** [1] to the pixel of an image as training data and the parameter of the model are learned by **EM-algorithm** [1]. The hidden or labeled image is constructed during of running EM-algorithm. In this paper, we used a numerically method of finding maximum a posterior estimation by using of EM-algorithm and Gaussians mixture model which is called **EM-MAP algorithm**. In this algorithm, we have made a sequence of the priors, posteriors and they then convergent to a posterior probability that is called the reference posterior probability. So Maximum a posterior estimation can be determined by this reference posterior probability which will make labeled image. This labeled image shows our segmented image with reduced noises.

A Markov random field (MRF), Markov network or undirected graphical model is a set of random variables. Markov random fields (MRFs) [10] have been widely used for computer vision problems, such as image restoration, image segmentation surface reconstruction and depth inference. Much of its success attributes to the efficient algorithms, such as Iterated Conditional Modes, and its consideration of both “data faithfulness” and “model smoothness.”

The **HMRF-EM** framework was first proposed for segmentation of brain MR images [11]. For simplicity, we first assume that the image is 2D gray-level, and the intensity distribution of each region to be segmented follows a Gaussian distribution. Given an image Y = (y1; : : : ; yN) where N is the number of pixels and each yi is the gray-level intensity of a pixel, we want to infer a configuration of labels X = (x1; : : : ; xN) where xi ϵ L and L is the set of all possible labels. In a binary segmentation problem, L = {0, 1}. According to the MAP criterion, we seek the labeling X^x which satisfies:

$$X^x = \text{argmax } \{P(Y|X, \theta), P(X)\} \text{-----------(1)}$$

Where, P(X) is the Gibbs distribution and the joint likelihood probability is:

$$P(Y|X, \theta) = \prod_i P(yi|X, \theta)$$
$$= \prod_i P(yi|xi, \theta xi) \ldots\ldots\ldots\ldots\ldots (2)$$

Where P(yi|xi; Θxi) is a Gaussian distribution with parameters $\theta xi = (\mu_{xi} \sigma_{xi})$.

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
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The major difference between MRF and HMRF is that, in HMRF, the parameter set θ is learned in an unsupervised manner. In a HMRF image segmentation problem, there is no training stage, and we assume no prior knowledge is known about the foreground/background intensity distribution. Thus, a natural proposal for solving a HMRF problem is to use the EM algorithm, where parameter set θ and label configuration X is learned alternatively.

EM ALGORITHM FOR PARAMETERS

Expectation maximization is done to minimize the likelihood function with respect to the parameters comprising the means and co-variances of the components and the mixing coefficient.

Step 1: Assume we have an initial parameter set $\theta^{(0)}$

Step 2: At the ith iteration, we have $\theta^{(t)}$, and we calculate the conditional expectation.

$$Q(\theta|\theta^{(t)}) = E[\ln P(X,Y|\theta)|Y,\theta^{(t)}]$$

$$=\sum_{X\in\chi} P(X|Y,\theta^{(t)})\ln P(X,Y|\theta)\ldots\ldots\ldots\ldots(3)$$

Where χ is set of possible configuration of labels

Step3:Mstep:

Now maximize $Q(\theta|\theta^{(t)})$ to obtain the next estimate:

$$\theta^{(t+1)}=\underset{\theta}{\operatorname{argmax}}\, Q(\theta|\theta^{(t)}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Let $\theta^{(t+1)} \longrightarrow \theta^{(t)}$ and repeat from E-step

Let G(z; θ_l) denote a Gaussian distribution function with parameters θ_l=(μ_l; σ_l)

$$G(z;\theta_l)=\frac{1}{\sqrt{2\pi\sigma_l^2}}\exp\left(-\frac{(z-\mu_l)^2}{2\sigma_l^2}\right)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

We assume prior probability can be written as

$$P(X)=\frac{1}{Z}\exp(-U(X))\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Where U(X) is the prior energy function. We assume that P(Y|X,)

$$P(Y|X,\theta) = \prod_i G(y_i, x_i)$$

$$= \prod_i G(y_i, \theta_{xi})$$

$$= \frac{1}{Z'}\exp(-U(Y|X))\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

By the help of these assumptions, HMRF-EM algorithm can be written as

1. Start with initial parameter set $\theta^{(0)}$
2. Then we calculate the likelihood distribution $P^{(t)}(y_i|x_i,\theta_{xi})$
3. By the help of current parameter we can estimate the labels by MAP estimation

$$X^{(t)} = \underset{X\in\chi}{\operatorname{argmax}}\{P(Y|X,\theta^{(t)})P(X)\}$$

$$=\underset{X\in\chi}{\operatorname{argmin}}\{U(Y|X,\theta^{(t)}) + U(X)\}\ldots\ldots\ldots\ldots\ldots(8)$$

Step 4: now by the help of Baye's rule we calculate the posterior (unobserved) distribution for all l element of L and all pixels y_i

$$P^{(t)}(l|y_i) = \frac{G(y_i;\theta_l)P(l|x_{Ni}^{(t)})}{P^{(t)}y_i}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Where $x_{Ni}^{(t)}$ is the neighborhood configurationof $x_i^{(t)}$,

$$P^{(t)}(y_i) = \sum_{l\in L} G(y_i;\ \theta_l)P(l|x_{Ni}^{(t)})\ldots\ldots\ldots\ldots\ldots(10)$$

$$P(l|x_{Ni}^{(t)})= \frac{1}{Z}\exp\left(-= \sum_{j\in Ni} Vc(l, x_t^{(t)}\right)\ldots\ldots\ldots(11)$$

Step 5: Now posterior distribution is used to update the parameters

$$\mu_l^{(t+1)} = \frac{\sum_i P^{(t)}(l|y_i)y_i}{\sum_i P^{(t)}(l|y_i)} \quad \ldots\ldots\ldots\ldots\ldots(12)$$

$$(\sigma_l^{(t+1)})^2= \frac{\sum_i P^{(t)}(l|y_i)(y_i-\mu_l^{(t+1)})^2}{\sum_i P^{(t)}(l|y_i)} \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

MAP ESTIMATION FOR LABELS

Maximum a posterior estimation (MAP) is is a mode of the posterior distribution. The MAP can be used to obtain a point estimate of an unobserved quantity on the basis of empirical data. It is closely related to Fisher's method of maximum likelihood (ML), but employs an augmented optim-ization objective which incorporates a prior distribution over the quantity one wants to estimate. MAP estimation can therefore be seen as a regularization of ML estimation.

In the EM algorithm we solve for X^x,

$$X^*=\underset{X\epsilon\chi}{argmin}\{U(Y|X,\theta) + U(X)\}\ldots\ldots\ldots(14)$$

With the given Y and θ, where the likelihood energy is,

$$U(Y|X,\theta)=\sum_i U(y_i|x_i,\theta)=\sum_i\left[\frac{(y_i-\mu_{xi})^2}{2\sigma_{xi}^2} + \ln\sigma_{xi}\right]..(15)$$

The prior energy function is:

$$U(X)=\sum_{c\epsilon C} V_C(X)\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

Where $V_c(X)$ is the clique potential (for estimation of noise level) and c is the set of all possible cliques.

We know that one pixel has four neighbors, then clique potential for pixel is defined as:

$$V_C(x_i, x_j)=\frac{1}{2}(1 - I_{xi,xj})\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

Where

$$I_{xi,xj}=\begin{cases}0 & if\ xi \neq xj\\ 1 & if\ xi = xj\end{cases}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

Now we have developed an iterative algorithm to solve (14)

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
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1. To start with, we have an initial estimate X(0), which can be from the previous loop of the EM algorithm.
2. Provided $X^{k,}$ for all $1 \le i \le N$, we find $x_i^{(k+1)} = \underset{l \in L}{\operatorname{argmin}}\{U(y_i|l) + U(X)\} \sum_{j \in N_i} V_C(l, x_j^{(k)})$ ....(19)
3. Repeat step 2 until $U(Y|X,\theta) + U(X)$ stops changing significantly or a maximum k is achieved.

GMM Based HMRF

A Gaussian Mixture Model (GMM) is a parametric probability density function represented as a weighted sum of Gaussian component densities. GMMs are commonly used as a parametric model of the probability distribution of continuous measurements or features. GMM parameters are estimated from training data using the iterative Expectation-Maximization (EM) algorithm

A Gaussian mixture model with g components can be represented by parameters:

$\theta_l=\{(\mu_{l,1},\sigma_{l,1},\omega_{l,1}),....,(\mu_{l,g},\sigma_{l,g},\omega_{l,g})\}$ ....(20)

Then the GMM now has a weighted probability

$G_{mix}(z;\theta_l)=\sum_{c=1}^{g} \omega_{l,c}\, G(Z; \mu_{l,c}, \sigma_{l,c})$ ....(21)

In the E-step, we determine which data should belong to which Gaussian component; in the M-step, we recomputed the GMM parameters.

Colour Image Segmentation.

The colour image segmentation and gray-level image segmentation difference is that, for a colour image, the pixel intensity is no longer a number, but for a 3-dimensional vector of RGB values:

$Y = (y_1, \ldots, y_N)$,

And $y_i = (y_{iR}, y_{iG}, y_{iB})^T$.

The parameters of a Gaussian mixture model now becomes

$\theta_l=\{(\mu_{l,1},\sigma_{l,1},\omega_{l,1}),....,(\mu_{l,g},\sigma_{l,g},\omega_{l,g})\}$ ....(22)

Also, the likelihood energy becomes

$U(Y|X,\theta)=\sum_i U(y_i|x_i, \theta)$

$=\sum_i[\frac{1}{2}(y_i - \mu_{xi})^T\Sigma_{xi}^{-1}(y_i - \mu_{xi})+\ln|\Sigma_{xi}|^{½}]$.......(23)

Color Image Segmentation result are shown in figure 1(b), 1(c) for K-Means clustering and GMM based HMRF.

Results and Discussion

We use above mentioned algorithm GMM based HMRF for color image segmentation.The result is shown in below figure 1,2 ,3 and 4.

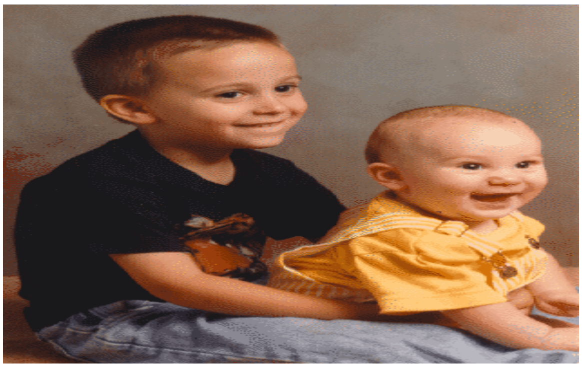

Figure 1(a) Original color image of Kids

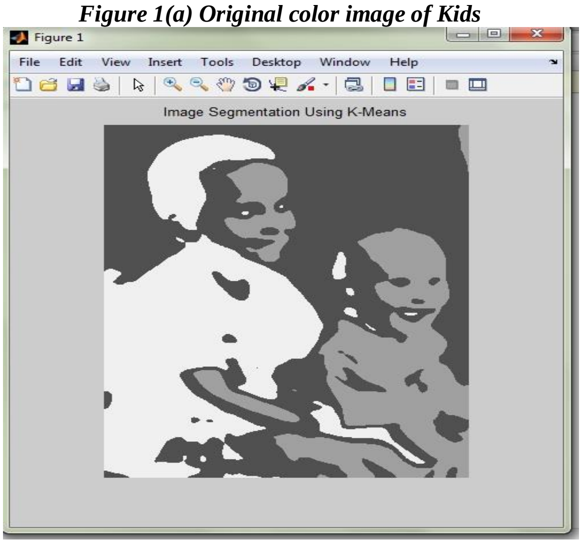

Figure1(b) Image segmentation using K-Means Clustering

Figure1(b) shows Image Segmentation using K-Means Clustering which does not give much smooth image. Some portion of an image like neck, face ,eaetc are not distinguishable from other parts of body.

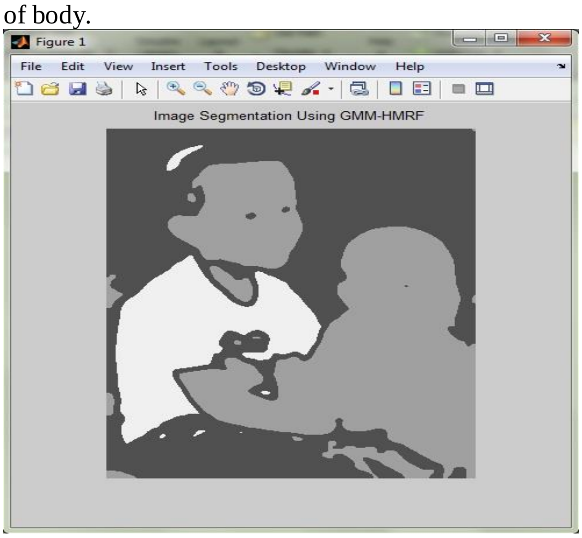

Figure1(c) Image Segmentation using GMM based HMRF

Figure1(c) shows an Image Segmentation using GMM based HMRF which gives moresmoother Image segmentation than K-Means clustering. In this the portion of an image which is not clearly

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Scientific Journal Impact Factor: 3.449
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visible in figure 1(b) using K-Means clustering is now clearly distinguishable from other portion of body parts.

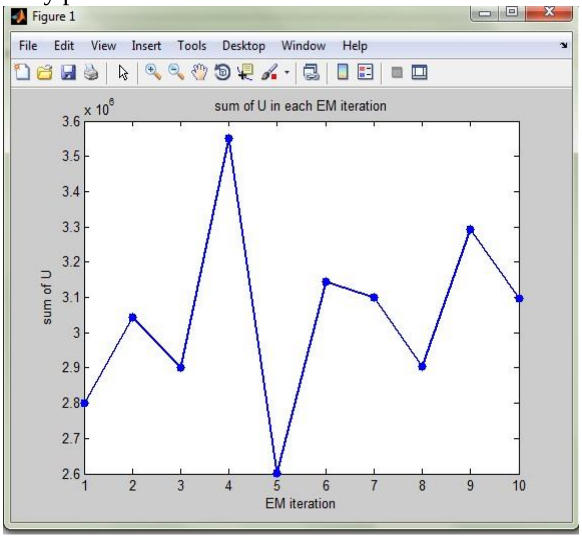

Figure1(e) shows sum of U in each EM Iteration

CONCLUSION

In this paper we have used hidden Markov random field, and its expectation-maximization algorithm. The basic idea of HMRF is combining "data faithfulness" and "model smoothness", which is very similar to active contours[4], gradient vector flow (GVF) [9], graph cuts [2],and random walks [3]. We have also used HMRF-EM framework with Gaussian mixture models, and applied it to color image segmentation. The algorithms are implemented in MATLAB R2013a.In color image segmentation experiments. We can see the HMRF segmentation results are much more smooth than the results of direct k-means clustering. This is because Markov random field imposes strong spatial constraints on the segmented regions, while clustering-based segmentation only considers pixel intensities.

Acknowledgements

First and foremost, we would like to thank Asst. Prof. Shanaz Aman for the enormous time and effort that she spent with us during this project. This work could not have been done without her excellent guidance, advice, encouragements and inspirations. She is a practical example of teacher as a friend, philosopher and guide.

We would also like to thank the members of Centurion University of Technology (CUTM), Electronics and Communication Engineering Department for their timely help during this project, those whose names are not mentioned here.

We would even like to thank all our friends, who have extended their hands for the completion of this project.

Last but not the least we convey special thanks to **Prof. Swarnaprabha Jena** of Electronics and Communication Engineering Department for providing encouragement.

REFERENCES

[1] StuthiEddula P Chandana, Ben SanthiSwaroop, "Implementing Gaussian Mixture model For Identifying Image Segmentation ", IJAIEM-2012,PP.29-35.

[2] Y. Boykov and G.F.Lea."Graph Cuts and Efficient ND Image Segmentation". International Journal Of Computer Vision,70(2):109-131,2006.

[3] L.Grady.Random walks for "Image Segmentation".IEEE Transactions on Pattern Analysis and Machine Intelligence,428(11):1768-1783,2006.

[4] M.Kass,A.Witkin and D.Terzopoulos Snakes: " Active Contour Models". International Journal Of Computer Vision,1(4):321-331,1988.

[5] A.Saxena, S.Chung and A.Ng, " 3-d depth reconstruction from a single still image". International Journal Of Computer Vision,76:53-69,2008.

[6] N.M.Vaidya and K.L.Boyer, "Discontinuity preserving surface reconstruction using stochastic differential equations.Computer Vision and Image Understanding",72(3):257-270,1998.[7] Q.Wang. "HMRF EM image: Implementation of the hidden markov random field model and its expectation maximizationalgorithm".arXiv:1207.3510[cs.CV], 2012.

[8] Q.Wang and K.L.Boyer, "The Active Geometric Shape Model: A new robust deformable shape model and its applications. Computer Vision and Image Understanding", 116(12):1178-1194,2012.

[9] C.Xu and J.L.Prince, "Snakes,shapes and gradient vector flow". IEEE Transactions on image processing, 7(3):359-369,1998.

[10] L.Zhang and Q. Ji, "Image Segmentation with a Unified Graphical Model. IEEE Transactions On Pattern Analysis and Machine Intelligence", 32(8):1406-1425, Aug. 2010.

[11] Y.Zhang, M.Brady and S.Smith. "Segmentation of Brain MR Images Throgh a hidden Markov Random Field Model and the expectation maximization algorithm". IEEE Transactions on Medical Imaging, 20(1):45-57, Jan.2001.

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

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